

Coexistence of itinerant ferromagnetism and a non-unitary superconducting state with line nodes: possible application to UGe_2

Jacob Linder,¹ Iver B. Sperstad,¹ Andriy H. Nevidomskyy,² Mario Cuoco,^{3,4} and Asle Sudbø¹

¹*Department of Physics, Norwegian University of Science and Technology, N-7491 Trondheim, Norway.*

²*Department of Physics and Astronomy, Rutgers University, Piscataway, N. J., 08854-8019*

³*Laboratorio Regionale SuperMat, CNR-INFM Salerno, Baronissi (Sa), Italy*

⁴*Dipartimento di Fisica E.R. Caianiello, Università di Salerno, Baronissi(Sa), Italy*

(Dated: Received April 23, 2008)

We construct a mean-field theory for itinerant ferromagnetism coexisting with a non-unitary superconducting state, where only the majority-spin band is gapped and contains line nodes, while the minority-spin band is gapless at the Fermi level. Our study is motivated by recent experimental results indicating that this may be the physical situation realized in the heavy-fermion compound UGe_2 . We investigate the stability of the mean-field solution of the magnetic and superconducting order parameters. Also, we provide theoretical predictions for experimentally measurable properties of such a non-unitary superconductor: the specific heat capacity, the Knight shift, and the tunneling conductance spectra. Our study should be useful for direct comparison with experimental results and also for further predictions of the physics that may be expected in ferromagnetic superconductors.

PACS numbers: 74.20.-z, 74.25.-q, 74.45.+c, 74.50.+r, 74.20.Rp

I. INTRODUCTION

The interplay between ferromagnetic (FM) and superconducting (SC) long range order microscopically coexisting in the same material has attracted much interest during the last decade due to the discovery of superconductivity in ferromagnetic metals, UGe_2 , URhGe , UCoGe , and possibly ZrZn_2 (see, however, Ref. 6).^{1,2,3,4,5} One possible route of investigation of such systems was adopted in early works^{7,8,9}, which assumed a conventional s -wave superconducting condensate residing in a ferromagnetic background caused by localized spins or aligned magnetic impurities. It was shown that below a critical value of the magnetic coupling, comparable to the superconducting gap Δ itself, superconductivity and magnetism were able to coexist. It was also suggested that a finite momentum pairing state, known as FFLO phase¹⁰, can appear in the presence of external magnetic field or intrinsic ferromagnetic order, and could thereby permit a larger threshold of the spin exchange energy to coexist with superconductivity.

On the other hand, it has been known since the early days of research on ^3He that alternative superconducting states, other than s -wave, can be favoured in a ferromagnetic background. The early theories of an equivalent phenomenon to occur in the solid-state have been formulated in the early 1980s¹¹, despite the absence of any experimental example of a ferromagnetic superconductor at the time¹². With the discovery of superconductivity in UGe_2 and ZrZn_2 , especially given that the same electrons are believed to participate both in ferromagnetism and SC, this latter scenario had to be taken seriously to explain the microscopic coexistence between the two phases. In particular, the very large hyperfine magnetic molecular field in these materials, measured¹³ *e.g.* with Mössbauer spectroscopy, far exceeds the Pauli limit. This excludes any possibility of singlet-pairing superconductivity.

We should note that although the latter statement is true in UGe_2 and other ferromagnetic superconductors, one may still ask whether in principle a singlet-type superconductivity can

coexist with ferromagnetism. Although some early theoretical studies¹⁴ indicated that the answer to this question may be affirmative provided FM is weak, a more careful analysis concluded¹⁵ that the coexistence state of spin-singlet pairing and ferromagnetism always turns out to be energetically unfavorable against the non-magnetic superconducting state even if a finite-momentum pairing (FFLO) state is considered. Later, it was proposed¹⁶ that the coexistence of metallic ferromagnetism and singlet superconductivity may be realized assuming that the magnetic instability is due to kinetic exchange. However, the coexistence of magnetism and spin-triplet superconductivity appears to be a more promising scenario, since the Cooper pairs may use their spin degree of freedom to align themselves with the internal magnetic field.

An experimental fact that is even more striking is that in all ferromagnetic superconductors known to date, the SC phase is only observed in a small part of the phase diagram otherwise occupied by ferromagnetism¹⁷, and it is the region where the magnetism appears to be at its weakest that SC sets in – on the boundary with paramagnetism when the Curie temperature is driven to zero (typically by applying pressure). This immediately raises the question of the microscopic origin of SC pairing, and whether ferromagnetic spin fluctuations play the role of a “glue” for Cooper pair formation very much as they do in superfluid ^3He . It is equally interesting what role the zero-temperature pressure-tuned phase transition plays in formation of superconductivity and whether notions involving quantum criticality (provided the phase transitions are second order) are necessary to explain the phenomenon.

Although there is no universal answer to this question yet and the research efforts, both experimental and theoretical, are focused on this issue, it is interesting to note that in both UGe_2 and ZrZn_2 the ferromagnetic phase transition as a function of pressure becomes 1st order as the “critical pressure” is approached at $T = 0$. One cannot therefore straightforwardly apply a theory of quantum criticality (be it the Hertz-Millis¹⁸ theory or one of its variations) given the absence of the quantum critical point as such. It is undeniable however that the

point where Curie temperature goes to zero is of crucial importance to the formation of the SC state.

Drawing further parallels between triplet-pairing FM superconductors and the superfluid ^3He , one may wonder whether different symmetries of the SC gap can occur, as is the case in the different phases¹⁹ of ^3He . For example, can the gap symmetry with point or line nodes be realized in the ferromagnetic superconductors? Very recently, experimental evidence has appeared which suggests that the answer is ‘yes’. Harada *et al.*²⁰ reported on ^{73}Ge nuclear-quadrupole-resonance experiments performed under pressure, in which the nuclear spin-lattice relaxation rate revealed an unconventional nature of superconductivity implying that the majority spin band in UGe_2 was gapped with line nodes, while the minority spin band remained gapless at the Fermi level.

Motivated by this, we present a mean-field model for coexisting ferromagnetism and spin-triplet superconductivity with a SC order parameter that displays line nodes in majority-spin channel and is gapless for minority spin. We first study the interplay between the magnetic and superconducting order parameters, and then proceed to make several predictions for experimentally relevant quantities: the specific heat capacity, Knight shift, and tunneling conductance. Let us briefly summarize our main results. We find that the low-temperature specific heat capacity C_V shows power-law behaviour (to be contrasted with the conventional exponential decay in the s -wave case), and that the gapless minority spins dominate the contribution to C_V at low temperatures, giving rise to a linear T -dependence. Also, the relative jump in C_V shows a strong dependence on the exchange splitting in the system. With regard to the Knight shift, we find that it is suppressed at $T = 0$ with increasing exchange splitting of the majority and minority spin bands when the external field is applied perpendicular to the spin of the Cooper pairs in the system. In general, however, it depends strongly on the orientation of the field with respect to the crystallographic axes of the compound, indicative of the triplet pairing in the system. Finally, the normalized tunneling conductance spectra show a strong directional dependence with respect to the orientation of the superconducting order parameter in reciprocal space, but change very little upon modifying the exchange splitting in the system. Our findings should be useful for comparison with experimental studies, and could lead to further insights as regards the nature of the superconducting order parameter.

This paper is organized as follows. We first describe the phenomenological framework to be used in this work in Sec. II. We then present our theoretical model in Sec. III, and provide the results of the self-consistent mean-field treatment at both zero and finite temperatures in Sec. IV A. We then proceed to make predictions for experimentally accessible quantities in Sec. IV B, using the self-consistently obtained results from Sec. IV A. We discuss our findings in Sec. VI, and summarize in Sec. VII. We will use boldface notation for vectors, $\hat{\cdot}$ for operators, $\hat{\cdot}$ for 2×2 matrices, and $\hat{\cdot}$ for 4×4 matrices.

II. PHENOMENOLOGICAL FRAMEWORK

The issue of coexisting ferromagnetism and superconductivity dates back to half a century ago when the celebrated FFLO state was predicted¹⁰ as a finite-momentum pairing state with real-space structure of the singlet SC order parameter that may develop under certain conditions close to the critical magnetic field H_{c2} . The conditions for the FFLO state are however very different from those observed in ferromagnetic superconductors such as UGe_2 . In particular, as has already been emphasised above, the magnetic molecular field felt by Cooper pairs inside the ferromagnet is many times larger¹³ than the Pauli limiting field necessary to destroy the singlet Cooper pairs. We shall therefore concentrate on triplet-type superconducting pairing.

Several remarks are in order. We note from the outset that the ferromagnetism observed in the Uranium compounds and in ZrZn_2 is itinerant, Stoner-like in its nature. We shall therefore not discuss the topic of localized magnetic moments that would have, among other things, provided a pair-breaking mechanism in accord with Abrikosov–Gor’kov theory²¹ of magnetic scattering. Here, we will assume that the same electrons involved in the spontaneous $\text{SU}(2)$ symmetry breaking associated with ferromagnetism, also participate in the $\text{U}(1)$ gauge symmetry breaking that characterizes a superconductor.

The idea of triplet pairing occurring between the same electrons that form the Stoner instability at the border of ferromagnetism goes back to Fay and Appel¹¹ (1980) who considered exchange of magnetic spin fluctuations as a microscopic mechanism for Cooper pairing. More recently, the problem has been revisited^{22,23,24,25,26} in the light of experimental findings in UGe_2 and other ferromagnetic superconductors.

In this paper, we shall take a phenomenological approach to superconductivity, leaving the intriguing and debated question of the microscopic mechanism for Cooper pairing aside. In particular, we shall consider systems where superconductivity appears at a lower temperature than the temperature at which onset of ferromagnetism is found. This is certainly the case experimentally and may be simply due to the fact that the energy scales for the two phenomena are quite different, with the exchange energy naturally being the largest. It may, however, also be due to the fact that superconductivity is dependent on ferromagnetism for its very existence. Such a suggestion has recently been put forth³².

A crucial issue to address in this context is whether superconductivity and ferromagnetism are phase-separated (such as *e.g.* solid and liquid phases coexisting at the melting point) or not. Fairly strong experimental evidence for non-phase-separated coexistence of ferromagnetism and superconductivity has recently been presented in UGe_2 ³³. However, even if such non-phase-separated coexistence is established, there still remains the issue of whether the superconducting order-parameters exhibits spatial variations, precisely due to its non-phase-separated coexistence with ferromagnetic order. One obvious candidate for such spatial variations³⁴ is a spontaneously formed Abrikosov vortex lattice, induced by the internal magnetization \mathbf{M} . As argued in Ref. 35, an impor-

tant factor with respect to whether a vortex lattice appears or not could be the magnitude of the internal magnetization \mathbf{M} . Specifically, Ref. 36 suggested that vortices may arise if $4\pi\mathbf{M} > \mathbf{H}_{c1}$, where \mathbf{H}_{c1} is the lower critical field. It is conceivable that a weak ferromagnetic state coexisting with superconductivity may give rise to a domain structure, in the absence of an external field, that is vortex-free. Therefore, we shall consider non-phase-separated coexistence of the FM and SC order parameters from here on, as have other studies³⁷. We will also leave the complications arising from the spatial variation of the superconducting order parameter originating with a putative spontaneously formed Abrikosov vortex lattice in the superconducting order parameter for future investigations.

Spin-triplet superconductors are characterized by a multi-component order parameter, which for the simplest case of the p -wave may be expressed in terms of three independent components of a \mathbf{d} -vector:

$$\mathbf{d}_{\mathbf{k}} = \left[\frac{\Delta_{\mathbf{k}\downarrow\downarrow} - \Delta_{\mathbf{k}\uparrow\uparrow}}{2}, \frac{-i(\Delta_{\mathbf{k}\downarrow\downarrow} + \Delta_{\mathbf{k}\uparrow\uparrow})}{2}, \Delta_{\mathbf{k}\uparrow\downarrow} \right]. \quad (1)$$

Note that $\mathbf{d}_{\mathbf{k}}$ transforms like a vector under spin rotations. In terms of the components of $\mathbf{d}_{\mathbf{k}}$, the order parameter itself is a 2x2 matrix that reads

$$\tilde{\Delta}_{\alpha\beta}(\mathbf{k}) \equiv \langle c_{\mathbf{k},\alpha} c_{\mathbf{k},\beta} \rangle = [i(\mathbf{d}_{\mathbf{k}} \cdot \boldsymbol{\sigma}) \sigma_y]_{\alpha\beta}, \quad (2)$$

where $\boldsymbol{\sigma}$ is the vector of Pauli matrices, and $c_{\mathbf{k},\alpha}^\dagger, c_{\mathbf{k},\alpha}$ are the usual electron creation-annihilation operators for momentum \mathbf{k} and spin α .

The superconducting order parameter is characterized as *unitary* if the modulus of the gap is proportional to the unity matrix: $(\tilde{\Delta} \cdot \tilde{\Delta}^\dagger) \propto \tilde{1}$. Written in terms of the vector $\mathbf{d}_{\mathbf{k}}$, this condition is equivalent to the requirement that $\langle \mathbf{S}_{\mathbf{k}} \rangle = 0$, where we have introduced the net magnetic moment (or spin) of the Cooper pair

$$\langle \mathbf{S}_{\mathbf{k}} \rangle \equiv i(\mathbf{d}_{\mathbf{k}} \times \mathbf{d}_{\mathbf{k}}^*). \quad (3)$$

The *unitary* triplet state thus has Cooper pairs with zero magnetic moment, whereas the *non-unitary* state is characterized by non-zero value of $\langle \mathbf{S}_{\mathbf{k}} \rangle \neq 0$. The latter effectively means that time-reversal symmetry is spontaneously broken in the spin part of the Cooper pairs²⁷. It is thus intuitively clear that having the spin of the Cooper pair aligned with the internal magnetic field of the ferromagnet can lower the energy of the resulting coexistence state. The above argument that the order parameter in the ferromagnetic superconductors must be non-unitary has been put forward by Machida and Ohmi²³, and others^{24,28}. Distinguishing between unitary and non-unitary states in ferromagnetic superconductors is clearly one of the primary objectives in terms of identifying the correct SC order parameter. To this end, recent studies have focused on calculating transport properties of ferromagnetic superconductors^{40,41,42,43,44,45}. There have also been investigations of identifying spin-triplet pairing in quasi-1D materials^{46,47,48,49}.

Finally, we note that inter-subband pairing is expected to be strongly suppressed in the presence of the Zeeman splitting between the \uparrow, \downarrow conduction sub-bands. In other words, only electrons within the same subband will form Cooper pairs (the

so-called *equal-spin pairing*) and we shall set $\Delta_{\uparrow\downarrow} = 0$ in what follows. Moreover, the requirement of non-unitarity of the order parameter then reduces to the requirement that the vector $\mathbf{d}_{\mathbf{k}}$ in Eq. (1) should have two non-zero components, i. e. $\Delta_{\uparrow\uparrow} \neq \Delta_{\downarrow\downarrow}$, which one would expect anyway in the presence of the Zeeman splitting between the two spin subbands. The spin of the Cooper pair is then $\langle S_z \rangle = \frac{1}{2}(|\Delta_{\uparrow\uparrow}|^2 - |\Delta_{\downarrow\downarrow}|^2)$ and is aligned along the magnetic field (z being the spin quantization axis).

III. THEORY

We consider a model of a ferromagnetic superconductor described by uniformly coexisting itinerant ferromagnetism and non-unitary, spin-triplet superconductivity. We write down a weak-coupling mean-field theory Hamiltonian with equal-spin pairing Cooper pairs and a finite magnetization along the easy-axis similar to the model studied in Refs. 25,26, namely

$$\hat{H} = \sum_{\mathbf{k}} \xi_{\mathbf{k}} + \frac{INM^2}{2} - \frac{1}{2} \sum_{\mathbf{k}\sigma} \Delta_{\mathbf{k}\sigma\sigma}^\dagger b_{\mathbf{k}\sigma\sigma} + \frac{1}{2} \sum_{\mathbf{k}\sigma} \left(\hat{c}_{\mathbf{k}\sigma}^\dagger \hat{c}_{-\mathbf{k}\sigma} \right) \begin{pmatrix} \xi_{\mathbf{k}\sigma} & \Delta_{\mathbf{k}\sigma\sigma} \\ \Delta_{\mathbf{k}\sigma\sigma}^\dagger & -\xi_{\mathbf{k}\sigma} \end{pmatrix} \begin{pmatrix} \hat{c}_{\mathbf{k}\sigma} \\ \hat{c}_{-\mathbf{k}\sigma}^\dagger \end{pmatrix}, \quad (4)$$

where $b_{\mathbf{k}\sigma\sigma} = \langle c_{-\mathbf{k}\sigma} c_{\mathbf{k}\sigma} \rangle$ is the non-zero expectation value of the pair of Bloch states. Applying a standard diagonalization procedure, we arrive at

$$\hat{H} = H_0 + \sum_{\mathbf{k}\sigma} E_{\mathbf{k}\sigma} \hat{\gamma}_{\mathbf{k}\sigma}^\dagger \hat{\gamma}_{\mathbf{k}\sigma}, \quad H_0 = \frac{1}{2} \sum_{\mathbf{k}\sigma} (\xi_{\mathbf{k}\sigma} - E_{\mathbf{k}\sigma} - \Delta_{\mathbf{k}\sigma\sigma}^\dagger b_{\mathbf{k}\sigma\sigma}) + \frac{INM^2}{2}, \quad (5)$$

where $\{\hat{\gamma}_{\mathbf{k}\sigma}, \hat{\gamma}_{\mathbf{k}\sigma}^\dagger\}$ are new fermion operators and the eigenvalues read

$$E_{\mathbf{k}\sigma} = \sqrt{\xi_{\mathbf{k}\sigma}^2 + |\Delta_{\mathbf{k}\sigma\sigma}|^2}. \quad (6)$$

It is implicit in our notation that $\xi_{\mathbf{k}} = \varepsilon_{\mathbf{k}} - E_F$ is measured from the Fermi level, where $\varepsilon_{\mathbf{k}}$ is the kinetic energy. The free energy is obtained through

$$F = H_0 - \frac{1}{\beta} \sum_{\mathbf{k}\sigma} \ln(1 + e^{-\beta E_{\mathbf{k}\sigma}}), \quad (7)$$

such that the gap equations for the magnetic and superconducting order parameters become²⁵

$$M = -\frac{1}{N} \sum_{\mathbf{k}\sigma} \frac{\sigma \xi_{\mathbf{k}\sigma}}{2E_{\mathbf{k}\sigma}} \tanh(\beta E_{\mathbf{k}\sigma}/2), \quad \Delta_{\mathbf{k}\sigma\sigma} = -\frac{1}{N} \sum_{\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'\sigma\sigma} \frac{\Delta_{\mathbf{k}'\sigma\sigma}}{2E_{\mathbf{k}'\sigma}} \tanh(\beta E_{\mathbf{k}'\sigma}/2). \quad (8)$$

Specifically, we now consider a model which should be of relevance to the ferromagnetic superconductor UGe₂, and possibly also for UCoGe and URhGe. In Ref. 20, it was argued

that the majority spin (spin-up in our notations) fermions were gapped and that the order parameter displayed line nodes, while the minority (spin-down) fermions remained gapless at the Fermi level in the heavy-fermion compound UGe_2 . An obvious mechanism for suppressing the superconducting instability in the minority-spin channel as compared to the majority-spin channel is the difference in density of states (DOS) at the Fermi level. Indeed, from Fig. 1 in Ref. 25 (see also Fig. 4 in Ref. 26), it is seen that the critical temperature for pairing in the minority-spin subband, T_c^\downarrow , is predicted to be much smaller than the critical T_c^\uparrow for the majority-spin subband, even for quite weak magnetic exchange splittings. Given the already quite low critical temperature T_c that is observed experimentally in ferromagnetic superconductors ($T_c \lesssim 1$ K), which we associate with T_c^\uparrow , we therefore conclude that it might indeed be very hard to observe experimentally the even smaller gap in the minority-spin subband. Therefore, it is permissible to only consider pairing in the majority-spin channel and neglect a small (if any) pairing between minority-spin electrons. In our notation this means setting $M \neq 0$, $\Delta_{\mathbf{k}\uparrow} \neq 0$, $\Delta_{\mathbf{k}\downarrow} = 0$.

We stress that the above statement, although intuitively attractive, may need further justification since we have so far neglected completely the spin-orbit interaction that is expected to be strong in Uranium based compounds, such as UGe_2 , URhGe and UCoGe . The effect of the latter would be to provide some effective coupling between majority and minority spin subbands and would probably lead to induced SC order parameter in minority spin channel. This issue is left for future study⁵⁰.

To model the presence of line nodes in the order parameter, we choose

$$\Delta_{\mathbf{k}\uparrow} = \Delta_{\bar{\mathbf{k}}_F\uparrow\uparrow} = \Delta_0 \cos \theta, \quad (9)$$

where $\bar{\mathbf{k}}_F$ is the normalized Fermi wave-vector, such that the gap only depends on the direction of the latter. This is the weak-coupling approximation. The above gap satisfies the correct symmetry requirement dictated by the Pauli principle, namely a sign change under inversion of momentum, $\theta \rightarrow \pi - \theta$. Here, θ is the azimuthal angle in the xy -plane. Our choice of this particular symmetry for the p -wave superconducting gap is motivated by the experimental results of Harada *et al.*²⁰. The $\cos \theta$ -dependence is also in accord with the results of Ref. 30, which showed that the majority band at the Fermi level for UGe_2 is strongly anisotropic with a small dispersion along the k_y -direction. We consider here a situation where the electrons are restricted from moving along the z -axis. The motivation for this is that, strictly speaking, it seems plausible that uniform coexistence of ferromagnetic and superconducting order should only be realized in thin-film structures where the Meissner (diamagnetic) response of the superconductor is suppressed for in-plane magnetic fields. The thin-film structure would then also suppress the orbital effect of the field. In a bulk structure, as considered in Ref. 14, we expect that a spontaneous vortex lattice should be the favored thermodynamical state³⁴, unless prohibited by a possible domain structure. Having said that, we point out that there is no firm experimental evidence for the presence

of such a vortex phase in ferromagnetic superconductors such as UGe_2 and ZrZn_2 , and we therefore do not exclude some mechanism that would instead stabilise a truly uniform coexistence of the SC and FM in these materials. It should be mentioned that uniform coexistence of ferromagnetism and superconducting order have also been speculated to occur in quasi-1D and quasi-2D materials such as $\text{RuSr}_2\text{GdCu}_2\text{O}_8$.⁵¹ In our model, the pairing potential may be written as

$$V(\theta, \theta') = -g \cos \theta \cos \theta', \quad (10)$$

where g is the weak-coupling constant. Conversion to integral equations is accomplished by means of the identity

$$\frac{1}{N} \sum_{\mathbf{k}} f(\xi_{\mathbf{k}\sigma}) = \int d\varepsilon N^\sigma(\varepsilon), \quad (11)$$

where $N^\sigma(\varepsilon)$ is the spin-resolved density of states. In three spatial dimensions, this may be calculated from the dispersion relation by using the formula

$$N^\sigma(\varepsilon) = \frac{V}{(2\pi)^3} \int_{\varepsilon_{\mathbf{k}\sigma} = \text{const}} \frac{dS_{\varepsilon_{\mathbf{k}\sigma}}}{|\hat{\nabla}_{\mathbf{k}} \varepsilon_{\mathbf{k}\sigma}|}. \quad (12)$$

With the dispersion relation $\xi_{\mathbf{k}\sigma} = \varepsilon_{\mathbf{k}} - \sigma IM - E_F$, one obtains

$$N^\sigma(\varepsilon) = \frac{mV \sqrt{2m(\varepsilon + \sigma IM + E_F)}}{2\pi^2}. \quad (13)$$

In their integral form, Eqs. (8) for the order parameters read

$$\begin{aligned} M &= -\frac{1}{4\pi} \sum_{\sigma} \sigma \int_0^{2\pi} \int_{-E_F - \sigma IM}^{\infty} d\theta d\varepsilon \frac{\varepsilon N^\sigma(\varepsilon)}{E_\sigma(\varepsilon, \theta)} \\ &\quad \times \tanh[\beta E_\sigma(\varepsilon, \theta)/2], \\ 1 &= \frac{g}{4\pi} \int_0^{2\pi} \int_{-\omega_0}^{\omega_0} d\theta d\varepsilon \frac{N^\uparrow(\varepsilon) \cos^2 \theta}{E_\uparrow(\varepsilon)} \tanh[\beta E_\uparrow(\varepsilon, \theta)/2]. \end{aligned} \quad (14)$$

For ease of notation, we also define

$$\begin{aligned} \Delta_\sigma(\theta) &= \begin{cases} \Delta_0 \cos \theta & \text{if } \sigma = \uparrow \\ 0 & \text{if } \sigma = \downarrow \end{cases}, \\ E_\sigma(\varepsilon, \theta) &= \begin{cases} \sqrt{\varepsilon^2 + \Delta_0^2 \cos^2 \theta} & \text{if } \sigma = \uparrow \\ \varepsilon & \text{if } \sigma = \downarrow \end{cases}. \end{aligned} \quad (15)$$

For the following treatment, we define $\tilde{M} = IM/E_F$, i.e. the exchange energy scaled on the Fermi energy. Moreover, we set $c = gN(0)/2$ to a typical value of 0.2 and $\tilde{\omega}_0 = \omega_0/E_F = 0.01$ as the typical spectral width of the bosons responsible for the attractive pairing potential. Finally, we define the parameter $\tilde{I} = IN(0)$ as a measure of the magnetic exchange coupling. As discussed below, only for $\tilde{I} > 1$ will a spontaneous magnetization appear in our model, in agreement with the Stoner criterion for itinerant ferromagnetism.

IV. RESULTS: MEAN-FIELD MODEL FOR COEXISTENCE

A. Zero temperature case

For zero-temperature, the superconducting gap equation reads

$$1 = \frac{g}{4\pi} \int_0^{2\pi} \int_{-\omega_0}^{\omega_0} d\theta d\varepsilon \frac{N^\dagger(\varepsilon) \cos^2 \theta}{\sqrt{\varepsilon^2 + \Delta_0^2 \cos^2 \theta}}. \quad (16)$$

Under the assumption that $\omega_0 \gg \Delta_0$, we obtain that

$$\frac{2}{c\sqrt{1+\tilde{M}}} = \ln\left(\frac{2\omega_0}{\Delta_0}\right) - \frac{1}{\pi} \int_0^{2\pi} d\theta \cos^2 \theta \ln|\cos \theta|. \quad (17)$$

which may be solved to yield the zero-temperature gap

$$\Delta_0 = 2.426\omega_0 \exp[-2/(c\sqrt{1+\tilde{M}})]. \quad (18)$$

By inserting Eq. (18) into the gap equation for the magnetization in Eq. (14), we have managed to decouple the self-consistency equations for M and Δ_0 . Numerical evaluation reveals that the gap equation for M is completely unaffected by the presence of Δ_0 , which physically means that the magnetization remains unaltered with the onset of superconductivity. This is reasonable in a model where the energy scale for the onset of magnetism is vastly different from the energy scale for superconductivity, such that by the time superconductivity sets in, the ordering of the spins essentially exhausts the maximum possible magnetisation.

The dependence of Δ_0 on \tilde{I} is shown in Fig. 1. The gap remains constant for $\tilde{I} \in [0, 1]$, which is a unitary phase. In the unitary phase, there is no reason for the minority spin band to remain ungapped when $M = 0$, and hence we would expect two gaps $\Delta_\uparrow = \Delta_\downarrow$ of equal magnitude for $\tilde{I} < 1$. Our model of gapping exclusively for the majority spin band is therefore justified only for $\tilde{I} > 1$, which is the regime we shall be concerned with throughout this article. The onset of a spontaneous magnetization for $\tilde{I} > 1$ is the well-known Stoner criterion for an isotropic electron gas, where the spin susceptibility may be written as⁵²

$$\begin{aligned} \chi(\mathbf{q}, \omega) &= \frac{\chi_0(\mathbf{q}, \omega)}{1 - I\chi_0(\mathbf{q}, \omega)}, \\ \chi_0(\mathbf{q}, \omega) &= N_0 \left(1 - \frac{\mathbf{q}^2}{12k_F^2} + i\frac{\pi\omega}{2v_F|\mathbf{q}|} \right), \\ |\mathbf{q}| &\ll 2k_F, \quad \omega \ll E_F. \end{aligned} \quad (19)$$

For a parabolic band, the static susceptibility is maximal for $\mathbf{q} = 0$ where

$$\chi(\mathbf{q} = 0, \omega = 0) = \frac{N_0}{1 - IN(0)} = \frac{N_0}{1 - \tilde{I}}. \quad (20)$$

The introduction of a ferromagnetic order is demarcated by the divergence of the susceptibility for $\tilde{I} = 1$, which is precisely Stoner's criterion for itinerant ferromagnetism. In the

absence of superconductivity, the self-consistency equation for the magnetization at $T = 0$ reduces to

$$h = -\frac{\tilde{I}}{3\sqrt{E_F}} \sum_{\sigma} \sigma [(E_F + \sigma h + \eta)^{3/2} - 2(E_F + \sigma h)^{3/2}], \quad (21)$$

where η is an upper energy cut-off determined by the bandwidth and $h = IM$ is the exchange splitting of the majority and minority bands. Since the energy scales for the magnetic and superconducting order parameter differ so greatly in magnitude, Eq. (21) is an excellent approximation even in the co-existent state (we have verified this numerically).

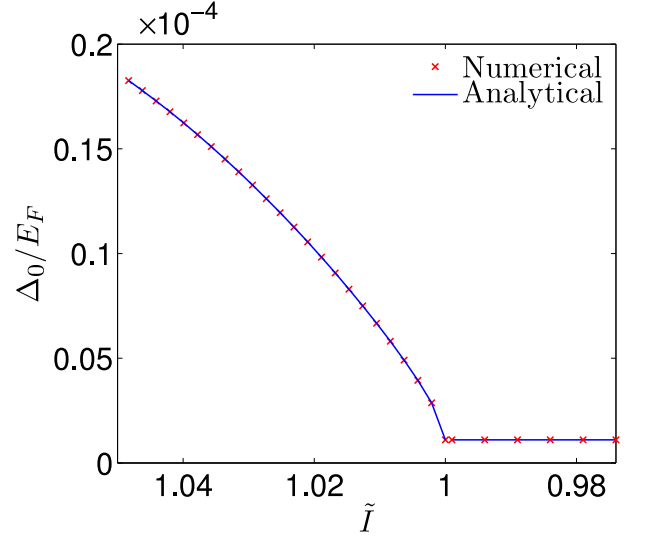


FIG. 1: (Color online) The gap-dependence on the ferromagnetic exchange interaction parameter $\tilde{I} = IN(0)$. The gap remains constant for $\tilde{I} \in [0, 1]$, corresponding to a unitary phase. The gap Δ_0 then starts growing with increasing \tilde{I} for $\tilde{I} > 1.0$, announcing the onset of a spontaneous magnetization. The analytical formula is based on Eq. (18).

B. Finite temperature case

The critical temperature for the superconducting order parameter is obtained in the standard way [setting $\Delta_0 = 0$ in Eq. (16)] to yield

$$T_c = 1.134\omega_0 \exp[-2/(c\sqrt{1+\tilde{M}})]. \quad (22)$$

In Fig. 2, we plot the temperature-dependence of the self-consistently obtained solution of Δ_0 and compare it to the analytical mean-field temperature dependence

$$\Delta_0(T) = \Delta_0(0) \tanh[\gamma \sqrt{T_c/T - 1}]. \quad (23)$$

The BCS result is $\gamma = 1.74$, but we find a better fit for our numerical results using $\gamma = 1.70$. Throughout the rest of this paper, we shall therefore make use of Eq. (23) with

$\gamma = 1.70$ to model the temperature-dependence of the gap for $\tilde{I} = \{1.01, 1.02, 1.03\}$, since the agreement is excellent with the full numerical solution. As in the zero-temperature case, we find that the gap equations in Eq. (14) may be completely decoupled also at finite temperature. We have verified that the gap equation for the superconducting order parameter has a unique non-trivial solution, which guarantees that the system will prefer to be in the coexistent state of ferromagnetism and superconductivity. The phase-diagram of the model we are

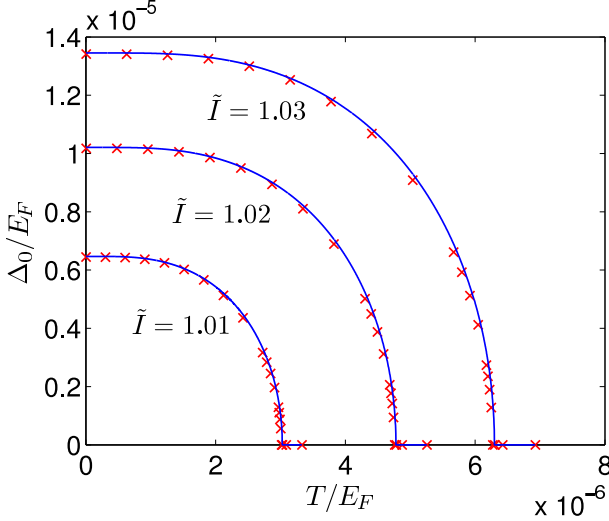


FIG. 2: (Color online) Self-consistently obtained solution for the superconducting gap Δ_0 (red symbols) compared to the analytical expression Eq. (23) with $\gamma = 1.70$ (blue lines), modelling a BCS-like temperature dependence.

considering may be obtained numerically and is shown in Fig. 3. As seen, a quantum phase transition may occur at $\tilde{I} = 1.0$, separating the 'unitary' superconducting state (see discussion in an earlier paragraph) from the ferromagnetic, non-unitary superconducting state. The critical temperature for the magnetic order parameter is orders of magnitudes larger than T_c for the superconductivity except for very close to $\tilde{I} = 1.0$. The increase in T_c in the non-unitary phase as compared to the unitary phase is a result of the increase in density of states with magnetization for the majority spin.

Experimentally, one often maps out the T - p phase diagram, where T is temperature and p is pressure. Note that the value of \tilde{I} may be controlled experimentally by adjusting the pressure on the sample. A change in pressure is accompanied by a change in the width of the electron bands, and therefore directly affects the density of states at the Fermi level: increasing the pressure on the samples reduces the density of states, and hence also the effective coupling constant \tilde{I} .⁵⁵ A notable feature in the phase diagram for UGe_2 as determined experimentally, is that superconductivity only appears in the ferromagnetic phase, and not in the paramagnetic phase.

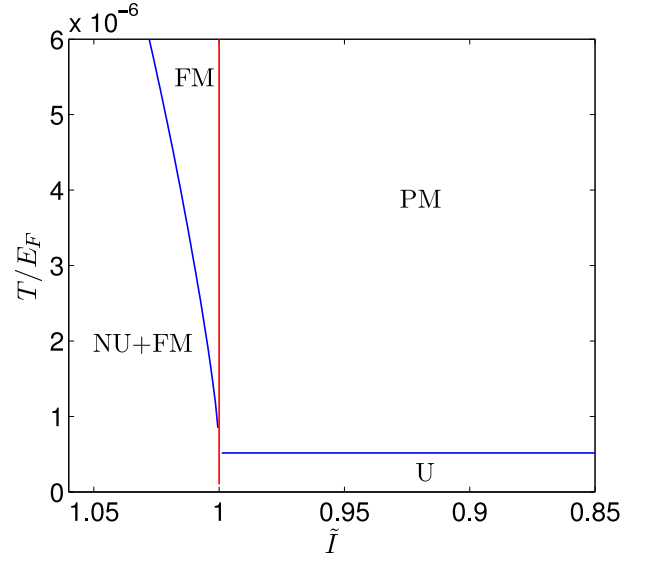


FIG. 3: (Color online) The phase-diagram of our model in the T - \tilde{I} plane. For $\tilde{I} > 1.0$, a spontaneous magnetization arises and allows for the possible uniform coexistence of ferromagnetism and triplet superconductivity. Note that decreasing \tilde{I} (going from left to right along the x-axis) corresponds to an increasing external pressure p . The abbreviations stand for non-unitary (NU), unitary (U), ferromagnetic (FM), and paramagnetic (PM).

V. RESULTS: EXPERIMENTAL PREDICTIONS

We next proceed to using the self-consistently obtained solutions from the previous section to make predictions for three experimental quantities that are routinely used to study superconducting condensates: specific heat, Knight shift, and tunneling conductance spectra. We first consider the normalized heat capacity, which is defined as

$$C_V = \frac{\beta^2}{8\pi} \sum_{\sigma} \int_0^{2\pi} \int_{-E_F - \sigma IM}^{\infty} d\theta d\varepsilon \frac{N^{\sigma}(\varepsilon)}{\cosh^2[\beta E_{\sigma}(\varepsilon, \theta)/2]} \times \left[E_{\sigma}^2(\varepsilon, \theta) - T \left(\Delta_{\sigma}(\theta) \frac{\partial \Delta_{\sigma}(\theta)}{\partial T} - \sigma \varepsilon I \frac{\partial M}{\partial T} \right) \right]. \quad (24)$$

Since the critical temperature of M is much larger than the critical temperature for Δ_0 in our model, we may safely neglect $\partial M / \partial T$ in the low-temperature regime. Consider Fig. 4 for a plot of the specific heat capacity using three representative values for \tilde{I} . The general trend with increasing \tilde{I} is an increase of the jump of C_V at $T = T_c$. The physical reason for this is that the majority spin carriers will dominate the jump in specific heat stronger when the exchange splitting between the bands increases, which is in agreement with the results of Ref. 26. Analytically, the relative jump in specific heat may be expressed as

$$\left(\frac{\Delta C_V}{C_V} \right) \Big|_{T=T_c} \sim \left(1 + \sqrt{\frac{1 - h/E_F}{1 + h/E_F}} \right)^{-1}. \quad (25)$$

It depends on the exchange splitting in the superconductor since the contribution from the majority spin carriers will tend

to dominate the specific heat when h increases. The low-temperature scaling with T bears witness of the line nodes in the gap, and is to be contrasted with the more rapidly decaying s -wave case. Also note that the minority spin fermions are in the normal state and give a significant contribution to the specific heat in form of a linear T -dependence at low temperatures. If both spin species were gapped with line nodes, one would expect a T^2 -dependence of the low temperature specific heat.

In the experimental study of the heat-capacity in UGe₂ conducted in Ref. 60, a peak of the heat-capacity associated with the superconducting transition was observed in a narrow pressure region $\Delta p \simeq 0.1$ GPa around p_x . Here, p_x is the pressure at which the superconducting transition temperature T_c shows a maximum value. Farther away from p_x , the heat capacity anomaly was smeared out. In particular, a substantial residual value of C_V/T was observed at $T \rightarrow 0$. The authors of Ref. 60 argued that neither the minority band density of states at the Fermi level nor the contribution from a self-induced vortex state would be appropriate to describe this residual value. Instead, it might stem from impurities that induce a finite density of states at the Fermi level. For an anisotropic superconductor like UGe₂, the residual value would be highly sensitive to such impurities. It is also clear that the observation of sharp peaks, similar to the ones we obtain in Fig. 4, depend strongly on the applied pressure on the superconductor, and in particular to how close it is to p_x .

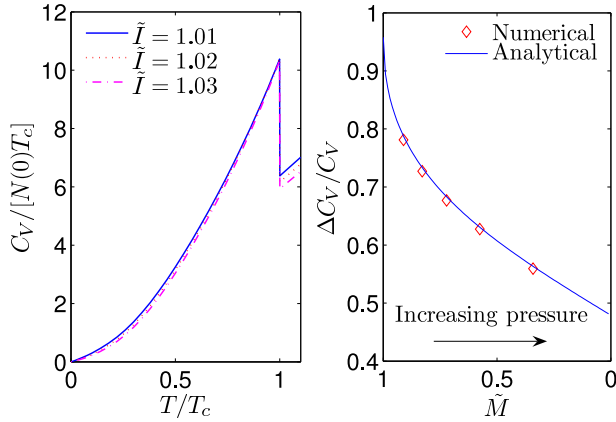


FIG. 4: (Color online) The left panel shows a plot of the specific heat capacity, using self-consistently obtained order parameters, for three different values of \tilde{I} . The right panel shows relative jump (superconducting vs. normal state) of the specific heat at the transition temperature as a function of the normalized exchange splitting between the spin bands, \tilde{M} . Numerically calculated values are shown in red, analytical result [Eq. (25)] using $\gamma \approx 1.70$ are shown in blue.

We next consider the spin susceptibility, making use of the standard formula³¹

$$\chi(\mathbf{q}, \omega) = -\frac{1}{2\beta} \sum_{\mathbf{k}, \omega_n} \text{Tr}\{\hat{\mathcal{G}}(\mathbf{k}, \omega_n) \hat{\mathcal{G}}(\mathbf{k} + \mathbf{q}, \omega + \omega_n)\}, \quad (26)$$

where $\hat{\mathcal{G}}$ is the matrix Green's function in particle-hole and spin-space, where $\omega_n = 2(n+1)\pi/\beta$ are fermionic Matsubara

frequencies. In the static ($\omega = 0$) and uniform ($\mathbf{q} = 0$) limit, Eq. (26) reduces to the Knight shift $\kappa \equiv \chi(0, 0)$. We define the normalized Knight shift as

$$\frac{\kappa}{\kappa_0} = \frac{\beta}{8\pi} \sum_{\sigma} \int_0^{2\pi} \int_{-E_F - \sigma IM}^{\infty} \frac{d\theta d\varepsilon N_{\sigma}(\varepsilon)}{\cosh^2[\beta E_{\sigma}(\varepsilon, \theta)/2]}. \quad (27)$$

The Knight shift is a measure of the polarizability of the conduction electrons in the compound, and serves as a highly useful probe to distinguish between singlet and triplet superconductivity. For a singlet superconductor, the total spin S of the Cooper pair is zero, and the Knight shift therefore vanishes at $T = 0$ since there are no quasiparticle excitations in the superconductor that may be polarized. The Knight shift vanishes regardless of the direction in which the external magnetic is applied for a singlet superconductor. For a triplet superconductor, this is quite different. The Knight shift now may be anisotropic in terms of the direction in which the magnetic field is applied. By means of the $\mathbf{d}_{\mathbf{k}}$ -vector formalism [see Eq. (1)], one may infer that the Knight shift is unaltered even for $T < T_c$ when $\mathbf{d}_{\mathbf{k}} \perp \mathbf{H}$, but is altered according to Eq. 27 when $\mathbf{d}_{\mathbf{k}} \parallel \mathbf{H}$. This is valid as long as the $\mathbf{d}_{\mathbf{k}}$ remains 'pinned' in the material due to *e.g.* spin-orbit coupling, and hence does not rotate with \mathbf{H} . Otherwise, the Knight shift would remain unaltered in any direction. Therefore, an anisotropic Knight shift is a strong signature of a vector character of the superconducting order parameter, and hence of a spin-triplet superconducting state.

In Fig. 5, we plot the Knight shift for several values of \tilde{I} . It is interesting to note that $\kappa(0)$ is reduced with increasing \tilde{I} . Physically, this may be understood by realizing that the density of states of ungapped minority spins at the Fermi level decreases as the exchange splitting between the majority- and minority bands increases. This results directly in a lower amount of polarizable quasiparticles, and hence the Knight shift becomes suppressed. For a fully polarized ferromagnet (half-metal), the Knight shift would therefore be identical to an s -wave singlet superconductor for an applied field satisfying $\mathbf{H} \parallel \mathbf{d}_{\mathbf{k}}$. This fact emphasizes the importance of measuring the spin susceptibility along several directions to identify the proper spin-symmetry of the superconductor.

As a final experimental probe for the interplay between ferromagnetism and superconductivity, we employ a Blonder-Tinkham-Klapwijk formalism²⁹ to calculate the tunneling between a normal metal and a ferromagnetic superconductor in the clean limit, using the self-consistently obtained values of the order parameters in the problem. From the results of Ref. 42, we find that the normalized tunneling conductance may be written as

$$\frac{G}{G_0} = \sum_{\sigma} \int_{-\pi/2}^{\pi/2} d\theta \cos \theta [1 + |r_{\sigma}^A(eV, \theta)|^2 - |r_{\sigma}^N(eV, \theta)|^2], \quad (28)$$

where G_0 is the normal-state conductance. Above, $r_{\sigma}^A(eV, \theta)$ and $r_{\sigma}^N(eV, \theta)$ designate the Andreev- and normal-reflection coefficient, respectively, and read

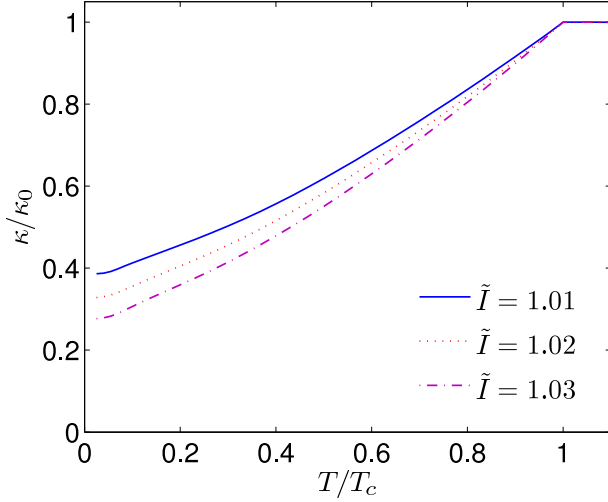


FIG. 5: (Color online) Knight shift for a ferromagnetic superconductor, using self-consistently obtained order parameters, for three different values of \tilde{I} . The field is here applied $\mathbf{H} \parallel \mathbf{d}_k$.

$$r_\sigma^N = -1 + \frac{2k_F \cos \theta [u_\sigma(\theta_{s+}^\sigma)u_\sigma(\theta_{s-}^\sigma)(\Upsilon_+^\sigma)^* + v_\sigma(\theta_{s-}^\sigma)v_\sigma(\theta_{s+}^\sigma)\gamma_\sigma(\theta_{s-}^\sigma)\gamma_\sigma^*(\theta_{s+}^\sigma)(\Upsilon_-^\sigma)^*]}{u_\sigma(\theta_{s+}^\sigma)u_\sigma(\theta_{s-}^\sigma)|\Upsilon_+^\sigma|^2 - v_\sigma(\theta_{s-}^\sigma)v_\sigma(\theta_{s+}^\sigma)\gamma_\sigma(\theta_{s-}^\sigma)\gamma_\sigma^*(\theta_{s+}^\sigma)|\Upsilon_-^\sigma|^2},$$

$$r_\sigma^A = \frac{4k_F \cos \theta q^\sigma \cos \theta_s^\sigma v_\sigma(\theta_{s+}^\sigma)u_\sigma(\theta_{s-}^\sigma)\gamma_\sigma^*(\theta_{s+}^\sigma)}{u_\sigma(\theta_{s+}^\sigma)u_\sigma(\theta_{s-}^\sigma)|\Upsilon_+^\sigma|^2 - v_\sigma(\theta_{s-}^\sigma)v_\sigma(\theta_{s+}^\sigma)\gamma_\sigma(\theta_{s-}^\sigma)\gamma_\sigma^*(\theta_{s+}^\sigma)|\Upsilon_-^\sigma|^2}. \quad (29)$$

We have defined $Z = 2mV_0/k_F$ as a measure of the barrier strength, where m is the quasiparticle mass, V_0 is the scattering strength of the barrier, and k_F is the Fermi momentum. Moreover, θ is the angle of incidence of incoming electrons from the normal side and we have implicitly incorporated conservation of group velocity and conservation of momentum parallel to the barrier, *i.e.* $k_F \sin \theta = q^\sigma \sin \theta_s^\sigma$. Finally, we have introduced

$$\Upsilon_\pm^\sigma = q^\sigma \cos \theta_s^\sigma \pm k_F \cos \theta \pm ik_F Z \quad (30)$$

and $\gamma_\sigma(\theta) = \Delta_\sigma(\theta)/|\Delta_\sigma(\theta)|$, $\theta_{s+}^\sigma = \theta_s^\sigma$, $\theta_{s-}^\sigma = \pi - \theta_s^\sigma$. In the quasiclassical approximation $E_F \gg (\Delta_0, \varepsilon)$, the wave-vectors read

$$k_F = \sqrt{2mE_F}, \quad q^\sigma = \sqrt{2m(E_F + \sigma IM)} \quad (31)$$

while the spin-generalized coherence factors are

$$u_\sigma(\theta_{s\pm}^\sigma) = \frac{1}{\sqrt{2}} \{1 + \sqrt{1 - (|\Delta_\sigma(\theta_{s\pm}^\sigma)|/E)^2}\}^{1/2},$$

$$v_\sigma(\theta_{s\pm}^\sigma) = \frac{1}{\sqrt{2}} \{1 - \sqrt{1 - (|\Delta_\sigma(\theta_{s\pm}^\sigma)|/E)^2}\}^{1/2}. \quad (32)$$

In Fig. 6, we plot the conductance spectra of a normal/ferromagnetic superconductor junction. By writing the gap as $\Delta = \Delta_0 \cos(\theta - \alpha)$, we allow for an arbitrary orientation of the gap with respect to the crystallographic axes. The

features seen in the conductance spectra are qualitatively different for $\alpha = 0$ and $\alpha = \pi/2$. In the first case, the electron- and hole-like quasiparticles entering the superconductor experience a constructive phase-interference which gives rise to the formation of a zero-energy state that is bound to the surface of the superconductor. The resonance condition for the formation of such zero-energy states is $\Delta(\theta) = -\Delta(\pi - \theta)$,⁵³ and the bound states are manifested as a giant peak in the zero-bias conductance⁵⁴. Note that such states exist even if the spatial depletion of the superconducting order parameter is not taken into account, which may be shown analytically⁵⁶. Taking into account the reduction the gap experiences close to the interface compared to its bulk value, is known to yield the same qualitative features as the usual step-function approximation, with the exception of additional, smaller peaks at finite bias voltages due to non-zero bound states⁵⁷. From Fig. 6, we see that the effect of increasing the exchange field amounts to sharper features in the conductance spectra. With increasing \tilde{I} , the zero-bias conductance peak becomes larger for $\alpha = 0$, while the dip structure for $\alpha = \pi/2$ becomes more pronounced. Physically, this may be understood by the increased contribution from majority spin carriers. The contribution from the minority spin carriers is constant for the entire low-energy regime, and leads to less pronounced features in the conductance. The effect of the barrier strength Z is seen in the left column of Fig. 6. For $\alpha = 0$, increasing Z leads to a higher peak at zero bias, while increasing Z suppresses the

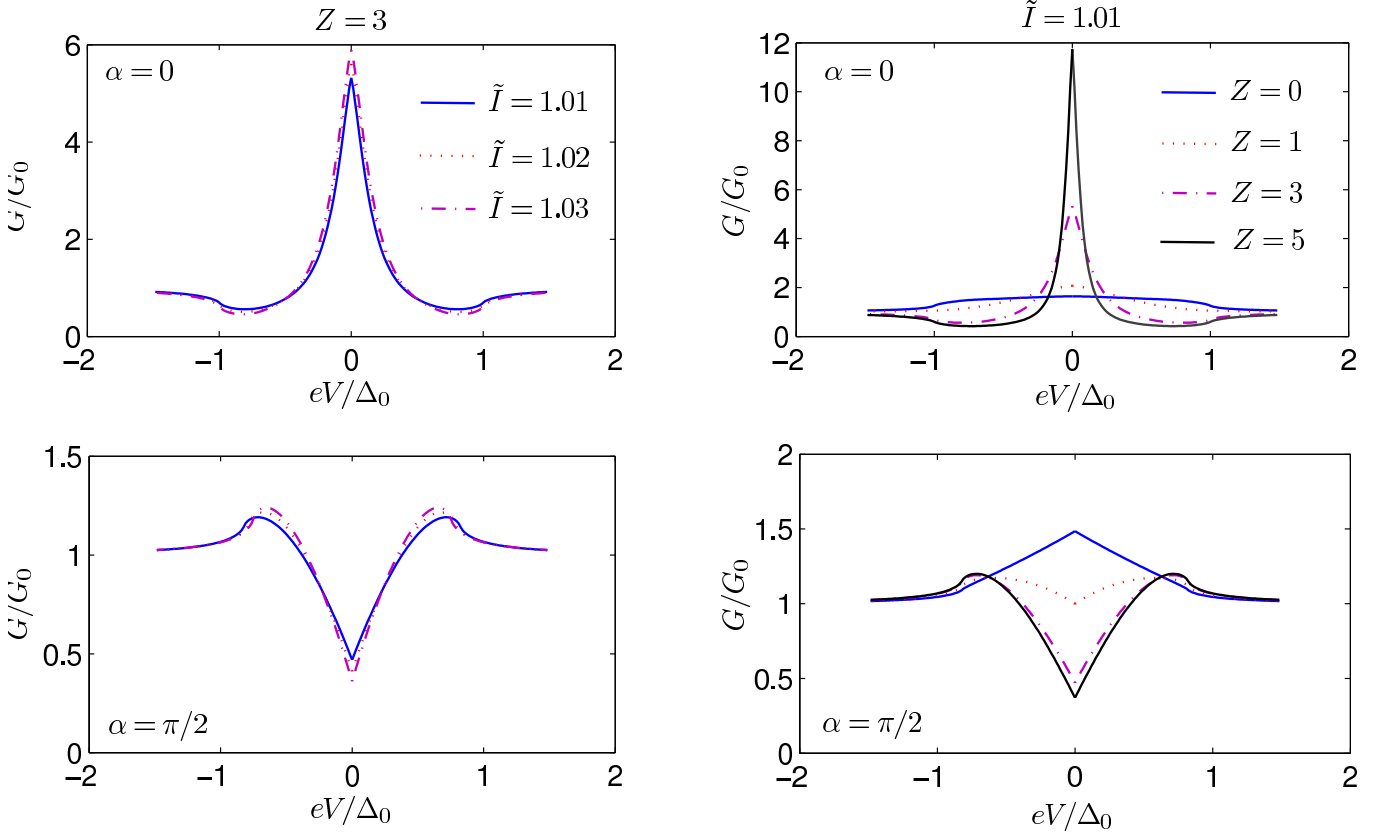


FIG. 6: (Color online) Plot of the tunneling conductance of a normal/ferromagnetic superconductor junction for $\alpha = 0$ and $\alpha = \pi/2$, using self-consistently obtained solutions at $T = 0$. In the left column, we fix the tunneling barrier strength $Z = 2mV_0/k_F = 3$ and plot the conductance for several values of the Stoner interaction \tilde{I} . In the right column, we fix $\tilde{I} = 1.01$ and plot the conductance for several values of Z .

conductance for $\alpha = \pi/2$.

It is also worth emphasizing the relation between the tunneling conductance and the bulk DOS of the superconductor. As is well-known, the conductance of a normal/*s*-wave superconductor junction in the tunneling limit approaches the DOS of the bulk superconductor²⁹. The same argument is valid for a $d_{x^2-y^2}$ -wave superconductor⁵⁴. One might be tempted to conclude that the tunneling conductance will always approach the bulk DOS of the superconductor in the strong barrier limit as long as there is no formation of zero-energy states. However, closer examination reveals that this is not necessarily so.

To illustrate this, we draw upon some results obtained in Ref. 59. In general, the conductance of an N/S junction in the tunneling limit may be written as

$$G(eV) \approx \frac{\int_{-\pi/2}^{\pi/2} d\theta_N \sigma_N \cos \theta_N \rho_S(eV)}{\int_{-\pi/2}^{\pi/2} d\theta_N \sigma_N \cos \theta_N}, \quad (33)$$

where σ_N is the normal-state conductance for a given angle of incidence θ_N and ρ_S is the surface DOS for the superconductor. In the absence of zero-energy states, the surface DOS coincides with the bulk DOS of the superconductor, i.e.

$\rho_S = \rho_0$, where

$$\rho_0(eV) = \int_{-\pi/2}^{\pi/2} d\theta_N \text{Re} \left\{ \frac{eV}{\sqrt{eV^2 - |\Delta(\theta_N)|^2}} \right\}. \quad (34)$$

An important consequence of the above equation is that the tunneling conductance may be interpreted as the expectation value of ρ_S with a weighting factor $\sigma_N \cos \theta_N$.

Let us now compare three different superconducting symmetries to illustrate the relation between the conductance and the DOS. We consider an *s*-wave, $d_{x^2-y^2}$ -wave, and p_y -wave symmetry, none of which feature zero-energy surface states (Fig. 8). Naively, one might therefore expect that the conductance should converge towards ρ_0 in the tunneling limit. However, it turns out that the weighting factor $\sigma_N \cos \theta_N$, which is peaked around $\theta_N = 0$, plays a major role in this scenario. In Fig. 7, we plot both the tunneling conductance $G(eV)/G_0$ and the bulk DOS ρ_0 for these three symmetries and fix $Z = 20$. We regain the well-known results that $G(eV)/G_0 \rightarrow \rho_0$ for large Z in the *s*-wave and $d_{x^2-y^2}$ -case. However, the conductance and DOS differ in the p_y -wave case.

The reason for the deviation between G/G_0 and ρ_0 in the p_y -wave case may be understood by consulting Fig. 8. As seen, the weighting factor is peaked around normal incidence

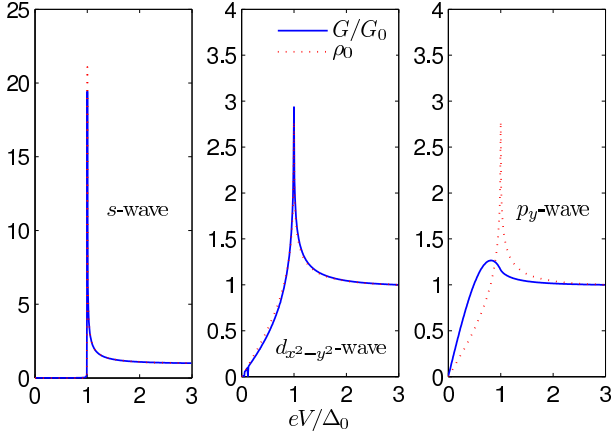


FIG. 7: (Color online) Plot of the normalized conductance G/G_0 and bulk DOS ρ_0 for three different symmetries of the superconducting state in the tunneling limit. Only in the p_y -wave case is there a difference between these two quantities.

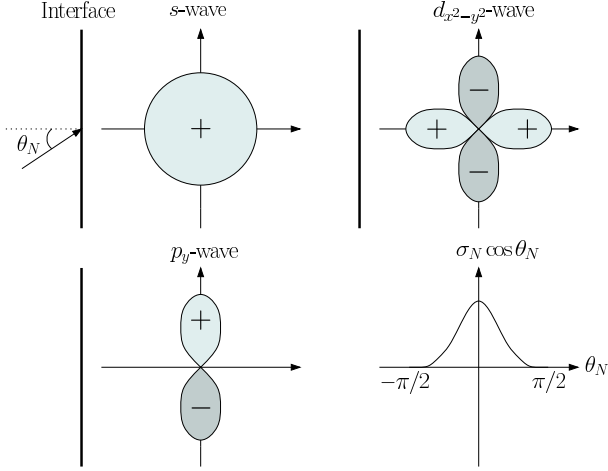


FIG. 8: (Color online) Illustration of the different symmetry states considered here and a qualitative sketch of the weighting factor $\sigma_N \cos \theta_N$.

$\theta_N = 0$. In the s -wave and $d_{x^2-y^2}$ -case, the gap magnitude is maximal at $\theta_N = 0$ and replacing the weighting factor in Eq. (33) with unity has little or no consequence. The situation is dramatically different in the p_y -wave case. Now, the gap magnitude is actually *zero* for normal incidence, and it is precisely this contribution that will dominate the integration over angles in Eq. (33). Therefore, replacing the weighting factor with unity, in order to obtain the DOS, has a non-trivial consequence in the p_y -wave case. This analysis illustrates how the conductance and bulk DOS in the absence of zero-energy states are not always the same in the tunneling limit. Note that the orientation of the interface with respect to the symmetry of the order parameter is crucial with regard to the measured conductance spectra and the surface DOS. For instance, even at $\alpha = \pi/4$ there is an appearance of a large zero-bias conductance peak for the p -wave pairing considered here, although the gap orientation does not satisfy the condition for perfect

formation of zero-energy states.

VI. DISCUSSION

We have discussed a mean-field model where itinerant ferromagnetism coexists with non-unitary, triplet superconductivity, with a gap that contains line nodes. The precise symmetry of the order parameter in the ferromagnetic superconductors UGe₂, URhGe, UCoGe is still under debate, although most experimental findings and theoretical considerations strongly point towards the realization of a triplet superconducting order parameter. It is plausible that such a superconducting order parameter is non-unitary, thus breaking time-reversal symmetry in the spin channel of the Cooper pair.

The orbital symmetry of the superconducting order parameter in ferromagnetic superconductors is a more subtle issue. In Ref. 26, a mean-field model for isotropic, chiral p -wave gaps in a background of itinerant ferromagnetism was constructed. In that work, pairing was assumed to occur both for majority- and minority-spins, resulting in for instance a double-jump structure in the specific heat capacity. An isotropic, chiral p -wave order parameter has a constant magnitude, which is favorable in terms of maximizing the condensation energy gained in the superconducting state. Assuming an isotropic density of states at the Fermi level and a separable pairing potential of the form $V_{\mathbf{k}\mathbf{k}'} = -g\lambda_{\mathbf{k}}\lambda_{\mathbf{k}'}$, the condensation energy gained at $T = 0$ in the superconducting state reads

$$E = -\frac{N(0)\Delta_0^2}{2}\langle|\lambda_{\mathbf{k}}|^2\rangle, \quad (35)$$

where Δ_0 is the maximum value of the gap and $\langle \dots \rangle$ denotes the angular average over the Fermi surface. This clearly shows the advantage of an isotropic gap $|\lambda_{\mathbf{k}}| = 1$. The general principle is well-known: the system prefers to have the Fermi surface as gapped as possible. However, factors such as spin-orbit pinning energy and lattice structure may conspire to prevent a fully isotropic gap. We also note that in our model, the ferromagnetic ordering enters at a much higher temperature than the superconducting order unless \tilde{I} is very close to unity. This is consistent^{1,2} with the experimental findings for the ratio between the critical temperatures for ferromagnetic and superconducting order, $T_c^{\text{FM}}/T_c^{\text{SC}}$, except for UCoGe where the ratio is $\simeq 3$.⁴

The experiments performed so far are indicative of a single gap, or at least a strongly suppressed second gap, in the ferromagnetic superconductors. For instance, no double-jump features have been observed in the specific heat capacity¹ for UGe₂. This warrants the investigation of a single-gap model, possibly with line nodes as suggested by Harada *et al.*²⁰ Theoretically, the absence of the SC gap in the minority spin subband can be justified by considering the effect of Zeeman splitting on the electronic density of states (see discussion in Sec. III and Ref. 25). In general, it should be possible to discern the presence of two gaps by analyzing specific heat or point-contact spectroscopy measurements, unless one of the gaps is very small.

Apart from this, another possible scenario, specific to UGe_2 , can be invoked to explain the observed gapless behaviour in the minority spin subband. This is the *meta-magnetic transition* that occurs inside the FM phase of UGe_2 and separates the two ferromagnetic phases with different values^{60,61} of magnetization M . The reason this meta-magnetic transition in UGe_2 is of great importance is because the specific heat measurements clearly indicate⁶⁰ that the maximum of superconducting T_c occurs not at the FM to PM transition, but at some lower pressure $p_x \approx 12$ kbar that coincides precisely with the meta-magnetic transition^{20,61}.

One can think of this transition as a point where the value of low-temperature magnetization M sustains a jump. While the microscopic origin of this transition is not known, an idea has been put forward⁶² that it may be due to a sharp change in the density of states (DOS) due to the existence of a double peak in its structure close to the Fermi level. What happens according to this scenario is that applied pressure makes the Fermi level “sweep through” the double-peak structure in the DOS, thereby sharply increasing the density of states in the majority spin channel. It follows from a simple Stoner instability argument that such an increase in the DOS would lead to a larger value of effective interaction $\tilde{I} \equiv IN(0)$ and thus higher magnetization M . But this also means that the ratio of the DOS in the two spin channels, N_\uparrow/N_\downarrow , sharply increases at the meta-magnetic transition. It follows from Eqs. (16, 18, 22) that the ratio between the SC gaps in the two spin subbands

$$\frac{\Delta_\downarrow}{\Delta_\uparrow} \propto \frac{T_c^\downarrow}{T_c^\uparrow} = \frac{\exp(-1/gN_\downarrow)}{\exp(-1/gN_\uparrow)} \quad (36)$$

thus becomes very small, justifying the assumption $\Delta_\downarrow = 0$ made in this work.

We note in passing that from an experimental point of view, a complication with UGe_2 is that the superconductivity does not appear at ambient pressure, in contrast to URhGe and UCoGe . The necessity of considerable pressure restricts the use of certain experimental techniques, and this is clearly a challenge in terms of measuring for instance conductance spectra of UGe_2 . Another experimental quantity which would be of high interest to obtain from for instance *ab initio* calculations, is the thermal expansion coefficient, which may be directly probed in high-pressure experiments⁵⁸.

We also underline that in our model the magnetism is assumed to coexist uniformly with superconductivity. Depending on the geometry of the sample, it is likely that the intrinsic

magnetization gives rise to a self-induced vortex phase. In a thin-film structure where the thickness t is smaller than the vortex radius λ , we expect that ferromagnetism and superconductivity may be realized in a vortex-free phase, similarly to a thin-film s -wave superconductor in the presence of an in-plane magnetic field. Further refinements leading to a more realistic model of a ferromagnetic superconductor should include the presence of spin-orbit coupling, which inevitably is present in heavy-fermion superconductors, in addition to the presence of vortices. Nevertheless, we believe that our model should capture important qualitative features of how the interplay between ferromagnetism and superconductivity may be manifested in experimentally accessible quantities. In particular, experiments on transport properties of ferromagnetic superconductors, such as the Josephson current and point-contact spectroscopy, would be of high interest to further elucidate the pairing symmetry realized in ferromagnetic superconductors.

VII. SUMMARY

In conclusion, we have constructed a mean-field theory of triplet superconductivity in the background of itinerant ferromagnetism, where the superconducting order parameter contains line nodes and the minority spin band remains ungapped at the Fermi level. We have solved the self-consistency equations for the order parameters in the problem, and find that ferromagnetism enhances superconductivity, while the ferromagnetism itself is virtually unaffected by the presence of superconductivity. We have made several predictions for experimentally accessible quantities: heat capacity, Knight shift, and tunneling conductance spectra. Our results may be helpful in the interpretation of experimental data, and could provide tools concerning the issue of identifying the pairing symmetry of ferromagnetic superconductors.

Acknowledgments

J.L. wishes to express his gratitude to Y. Tanaka at Nagoya University for his hospitality, where parts of this work were completed. N. Sato is also thanked for very useful discussions. This work was supported by the Norwegian Research Council Grants No. 158518/431 and No. 158547/431 (NANOMAT), and Grant No. 167498/V30 (STORFORSK).

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